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Infrared Emissions from Turbofans with High Aspect Ratio Nozzles

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A mixed flow turbofan is analyzed from the point of view of i.r. emission characteristics. A simple absorption coefficient model of the core flow gas displays the potential effectiveness of high aspect ratio nozzles as a variable in the design of aircraft engine installations for low i.r. signature. The simplicity of the model described and the limited view perspectives used to assess the signature restrict the usefulness of the results to that of guiding preliminary design. In general, more precise solutions are complex and depend on specification of a relatively large number of independent variables to describe the source—seeker geometry, weather, etc. The level of effort to carry out more accurate analysis may be inconsistent with the preliminary design process where simple criteria such as those described here may be sufficient to select the best of several candidate designs. The influence of cycle parameters is assessed using an *n*th power dependence of radiation on temperature and a simple mixing model to estimate the core length. The analysis shows that cycle parameters which improve cycle efficiency and thus fuel consumption also reduce i.r. emissions. Bypass ratio near unity gives low i.r. signature for both optically thin and thick spectral regions.

Nomenclature

A = area
AR = aspect ratio $\equiv W_0/H_0$
B = black body spectral radiant intensity, W/cm^2-sr-μ
C_p = specific heat
D = emitting gas slab thickness
F = thrust
H = jet core height
h = fuel heating value
I = spectral emission intensity, W/cm^2-sr-μ
J = spectral radiative power, W/sr-μ
K = proportionality constant [Eq. (21)]
k = optical absorption coefficient
M = Mach number
$\dot{m}$ = mass flow rate
n = temperature dependence parameter
p = pressure
R = spectral volumetric emissive power, W/cm^3-sr-μ
S = specific fuel consumption
T = temperature
u = flow velocity
V = core volume
W = jet core width
x = streamwise coordinate
y = coordinate normal to x
β = bypass ratio
γ = specific heat ratio
θ = compressor inlet total temperature to ambient temperature ratio
θ_t = turbine inlet total temperature to ambient temperature ratio
τ = temperature ratio
Δ = reference temperature parameter, see Eq. (29a)

Subscripts

ab = afterburner
C = compressor
F = fan
T = turbine or turbine inlet
t = total or stagnation
tot = total engine air flow (see Fig. 4)
0 = initial core dimensions (on W, H) and freestream static conditions (on T)
2 = compressor inlet
3 = compressor outlet
4 = turbine inlet
5 = turbine outlet
6 = mixer outlet
7 = afterburner outlet

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I. Introduction

THE vulnerability of military aircraft to detection and to lock-on by infrared seeking missiles is, in part, related to the thermal emission characteristics of the heated combustion products of the jet plume. The radiation signature seen by an observer viewing an airplane from an aft perspective is determined by the contributions of a number of emission elements. These are the core and its gasdynamic structure, the mixing region behind the core, the hot parts of the engine and nozzle, as well as reflections from the engine interior.

The core is the unmixed engine exhaust gas which may be described as a semitransparent volume of CO_2 and H_2O bounded by a cone (if the nozzle is round) radiating at the static temperature of the gas. The transparency depends on the optical thickness which, in turn, depends strongly on wavelength. This core radiation source makes the most important and the most persistent contribution to the radiation signature.[1] As a consequence, the other contributions will not be considered for the development of the simplified preliminary design model to be described here. These contributions may be designed to be sufficiently small for their influence to be negligible for design purposes. The reasons or methods relevant to the problem of reducing this influence are described below.

The core radiation is influenced by gasdynamic structure if the gas is improperly expanded because of the shock waves imbedded in the flow. Such shocks give rise to axially nonuniform temperature distributions and hence contribute to the total radiation.[2] Under appropriate circumstances a contribution by core flow structure may be reduced to insignificance by designing sufficient area variation flexibility, length, etc., into the nozzle, provided the designer can afford the weight and cost penalties.

The mixing region is that portion of the flow where transport processes are at work to smooth variations in the descriptive variables of the jet and external flow. Hence a mixing region will be found adjacent to the hot core as well as following the hot jet core. In the adjacent shear layer region the velocity and temperatures will vary in a smooth way when such measurements are time averaged. Since the emitted radiation from a gas is strongly temperature dependent [T^n, n=4-5. Ref. 3 gives 4680(°R)/T+1.22] only the core will contribute significantly. Hence as far as the adjacent shear layer is concerned, one may think of the core as having a slightly larger effective transverse dimension because of the adjacent shear layer, although absorption effects in that layer may be important.

The mixing region behind the core is nonuniform in both the axial and transverse directions. The temperature decreases with distance from the nozzle and the gas volume is very large. To argue that the mixing plume behind the core contributes little to the source radiation seen by an observer, it is only necessary to note that the radiative power (T^n) per unit volume decreases faster than the volume as one proceeds away from the end of the core.

Hot parts emissions and reflections may be reduced to minimum levels by appropriate geometric design of the turbine exit and the nozzle.[4] These elements are therefore design-dependent and will not be considered further here.

In what follows, attention will be restricted to the question of the influence of the nozzle aspect ratio and the turbofan cycle on the radiation from only the hot core and this only from a 90 deg aspect angle (normal to flight) because this approach minimizes the complexity of the problem and exposes the results sought most clearly. The attack/detection problem from a 90 deg aspect angle is but one of a number of threat scenarios and care must be taken when using the results described here that the 90 deg aspect angle view is at least representative of the full range of possibilities.

The emitted radiation in the problem under consideration is the thermal emission from exhaust gases in the range 800-1600°F (700-1150 deg K). The constituents in the exhaust gas which contribute to the radiation are primarily CO_2, H_2O, CO, and particulates. In the discussion to follow we will concentrate primarily on the contribution made by CO_2 because it is the strongest radiator and its absorption by atmospheric CO is modest and independent of weather and climate. Water vapor, on the other hand, is also an emitter of radiation but, because it is present in natural air in widely varying proportions, water vapor radiation is not often used for the detection of aircraft by missiles. The presence of CO and particulates can be minimized by the choice of fuel and by the design of the combustion process.

II. Influence of Nozzle Aspect Ratio

Consider a "two-dimensional" nozzle as shown in Fig. 1 where an exhaust gas jet exits the nozzle with velocity u_6 as shown, mixing with a parallel freestream of velocity u_0. The nozzle dimensions are W_0 and H_0 and the two viewing angles considered are as indicated. The mixing process between the exhaust gas and the freestream leads to determination of the core length L, as shown. The hot core is thus pyramidal in shape with a ridge in the wider dimension. A detector viewing this core from the s (side) or v (vertical) directions will see either a trapezoid or triangular area as shown in Fig. 2. To eliminate the atmospheric environment as a parameter in the problem, one may calculate a source intensity which is the radiation intensity reaching a hypothetical detector far away with no atmospheric absorption. This allows competing designs to be compared on an equal basis, since the atmospheric environment generally affects the source intensity of the various designs equally. This intensity is given by the volumetric emissive power per unit solid angle, R, which is the number of photons per unit volume per steradian per micron emitted multiplied by the probability that a photon is not absorbed in traversing the gas from its point of origin to the edge of the emitting volume.[5] Thus viewing the gas sample along the direction y, the differential contribution to the intensity reaching the edge from an element of length dy, at a depth y, would be dI=Rdy were it not for the fact that a photon from that element has only an exp-ky probability of reaching the edge. Thus

$$dI = Re^{-ky}dy \tag{1}$$

(All symbols are summarized in the nomenclature section.) While the use of an absorption coefficient is approximate, it is qualitatively correct and justified by its mathematical simplicity. The general conclusions regarding overall parametric behavior of the variables are not likely to be impacted significantly by this assumption. The intensity I, the volumetric emissive power R, and the absorption coefficient k are all functions of the wavelength. In the following, the customary subscripts on these quantities to denote explicit dependence on wavelength will be omitted for clarity.

With no absorption in the space between the observer and the gas slab, R and k uniform (uniform gas temperature and

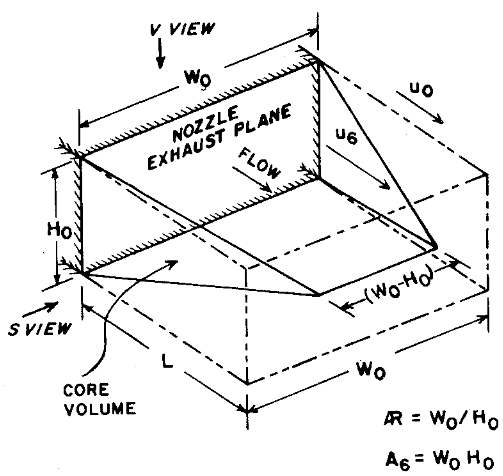

Fig. 1 Schematic of a rectangular two-dimensional nozzle showing the volume of hot core gas.

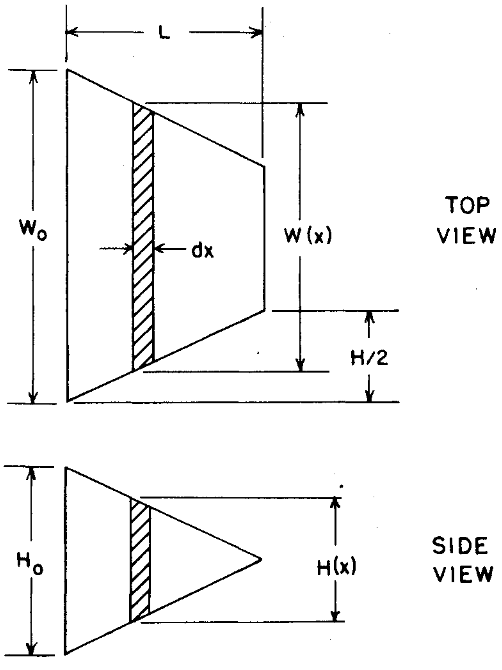

Fig. 2 Side and vertical views of core volume.

composition), one may integrate Eq. (1) to obtain the radiation from a gas slab of thickness D in the y direction and obtain

$$I = (R/k)(1 - e^{-kD}) = B(1 - e^{-kD}) \tag{2}$$

The ratio R/k is the black body spectral radiant intensity B. The spectral radiation intensity reaching an observer is B, when kD is very large or RD when kD is very small. We note therefore that the emission energy depends on the surface area *if kD is large* and on the emission volume *if kD is small.*

One may take the viewpoint that kD may be treated as a parameter to assess the influence of gas slab geometry independent of spectral characteristics of gas. Normally, parts of the spectrum will have kD small (optically thin) while in others kD will be large (optically thick). The total energy reaching an observer will be an integration of the spectral intensity over the wavelength band where the detector is responsive. In such an integration, each differential contribution is attenuated by the atmosphere between the source and the observer.

The core length L in Fig. 1 is determined by the smallest transverse dimension of the nozzle and the relative velocity difference between the freestream (u_0) and the jet exhaust (u_6). The following expression is a good representation[6,7] of the core length in terms of the minimum transverse dimension, H_0:

$$L \propto H_0 (u_6 + u_0)/(u_6 - u_0) \tag{3}$$

For the preliminary design information sought here, the accuracy of this equation for describing the core length is quite satisfactory in the density and Mach number range of interest and certainly consistent with the other assumptions made in this analysis.[8]

The problem of determining the source radiation from the core shown in Figs. 1 and 2 thus depends on the core dimensions which include the nozzle exit area, $W_0 H_0$, as well as the nozzle aspect ratio, W_0/H_0. We note that the two views, s and v, require an association of the appropriate core dimension with the optical depth. The differential volume element is given by

$$dV = H(x)\, W(x)\, dx \tag{4}$$

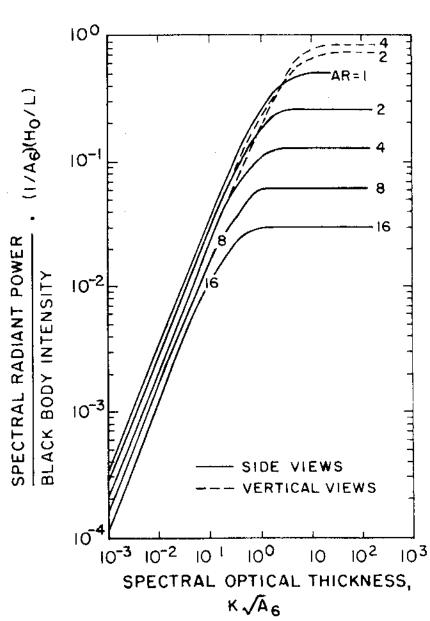

Fig. 3 Radiant energy seen by an observer viewing the hot core from side and vertical directions as a function of optical thickness and nozzle aspect ratio.

and the core height and width are assumed to vary linearly with increasing distance from the nozzle. In this model, a nozzle of aspect ratio 1 is a square nozzle which might, in practice, be round for a realistic design.

For the sideview, the optical depth of each element of length dx is $W(x)$ and the power emitted normal to the stream direction is

$$I_s = \int_0^L B H(x)\,(1 - e^{-kW(x)})\, dx \tag{5}$$

or integrating

$$\frac{I_s}{B}\frac{1}{H_0 L} = \frac{1}{2}\left[\left(\frac{\mathit{AR}}{kW_0}\right)^2 + \frac{\mathit{AR}}{kW_0}\right]e^{-kW_0} - \left(\frac{\mathit{AR}}{kW_0}\right)^2 e^{kW_0(1/\mathit{AR} - 1)} \tag{6}$$

where AR is the aspect ratio, W_0/H_0.

In a similar way the vertical view spectral energy emitted is

$$I_v = \int_0^L B W(x)\,(1 - e^{-kH(x)})\, dx \tag{7}$$

which when integrated yields

$$\frac{I_v}{B}\frac{1}{W_0 L} = 1 - \frac{2}{\mathit{AR}} - \left(\frac{\mathit{AR}}{(kW_0)^2} - \frac{\mathit{AR}}{kW_0}\right) \times (1 - e^{-kW_0/\mathit{AR}}) + \frac{1}{kW_0} \tag{8}$$

The spectral intensity for the two views may be written in terms of two parameters; one of which is the aspect ratio and the other is an optical depth based on characteristic nozzle dimension. We choose the parameter $k\sqrt{A_6} = k\sqrt{W_0 H_0}$ as the appropriate parameter. As expected, the results scale linearly with nozzle area and the gas velocity ratio u_6/u_0 through H_0/L as given by Eq. (3). The nondimensional radiant intensities for the two views are plotted in Fig. 3. Note that for small optical depth, the volume determines the total emitted radiative power which is isotropic for all aspect ratios. Increasing the aspect ratio under these conditions serves to reduce the radiation from that which would be emitted if the aspect ratio were unity. In particular, an aspect ratio of 8 is required to reduce the emitted radiation by a factor of 2. In the limit of $k\sqrt{A_6}$ much less than unity Eq. (8) reduces to

$$\frac{I}{B}\frac{1}{A_6 (L/H_0)} = k\sqrt{A_6}\,\frac{3\mathit{AR} - 1}{6\mathit{AR}^{3/2}} \tag{9}$$

This result is independent of viewing direction.

If the optical thickness is large, the radiation becomes less isotropic, with that seen from the vertical direction significantly larger than that which is seen from the side direction. In the large $k\sqrt{A_6}$ limit the emitted power reduces to

$$\frac{I_s}{B}\frac{1}{A_6 (L/H_0)} = \frac{1}{2\mathit{AR}}$$
$$\frac{I_v}{B}\frac{1}{A_6 (L/H_0)} = 1 - \frac{1}{2\mathit{AR}} \tag{10}$$

Thus it is clear that any appreciable reduction in i.r. emissions from the core at a given temperature can only be obtained in the horizontal (side) viewing direction. The radiation in the vertical direction increases with decreasing aspect ratio to a limiting value.

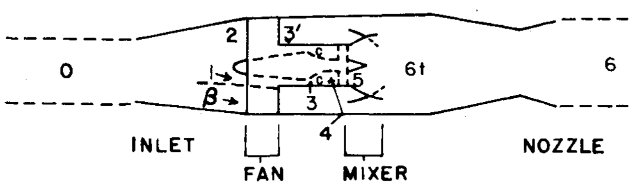

Fig. 4 Engine schematic and station numbering system.

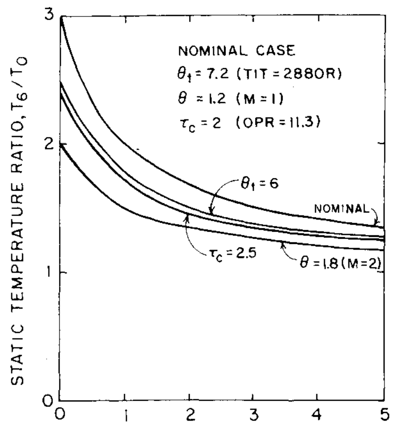

Fig. 5 Variation of engine exhaust gas temperature with bypass ratio. Note OPR is the overall pressure ratio of the compressor, and TIT is the turbine inlet temperature/T_0(= 400°R).

III. Cycle Considerations

From the discussion in Sec. II, it is clear that varying nozzle aspect ratios can play an important role in determining gas core source radiation which depends on the spectral optical depth, the nozzle size, and the length of the core. These last two parameters are influenced by the choice of the thermodynamic cycle of the turbofan engine. In the following, the role played by the choice of the thermodynamic cycle chosen for the engine, which influences both the exhaust gas temperature and the jet velocity, will be considered. Note that the thrust level as well as the radiation are determined by the nozzle exit area A_6.

For purposes of this analysis we assume that the aircraft utilizing the engine to be analyzed here is flying at a specified Mach number. The flight is assumed to be steady. The altitude will scale quantities like the emitted radiative power directly through the number density of active molecules[9,10] and hence needs not to be considered separately.

The engine cycle to be analyzed is the ideal turbofan with forced internal mixer after which afterburning may be carried out. The ideal cycle uses ideal air as a working fluid throughout, has zero pressure losses through the inlet, burner, mixer, and afterburner duct, and follows isentropic compression and expansion processes. Furthermore, it is assumed that the nozzle is correctly expanded. Note that the turbojet is a turbofan with zero bypass ratio. The ideal turbofan with matched fan and primary pressures is parametrically described by the turbine inlet temperature, the overall compressor pressure ratio and the bypass ratio. Its characteristics are easily analyzed and will display infrared emission behavior similar to that of an engine with realistic component efficiencies. The engine and the flow station numbering system is shown schematically in Fig. 4. Following analyses similar to Refs. 11 and 12, we may write a relationship between total temperatures at the various flow stations implied by a work balance between turbine, compressor and fan. The following terms may be defined: θ_t = turbine inlet (TIT) to ambient temperature ratio = T_{t4}/T_0; $\theta = T_{t2}/T_0 = 1 + [(\gamma - 1)/2]M_0^2$; M_0 = flight, Mach number; β = bypass ratio; and

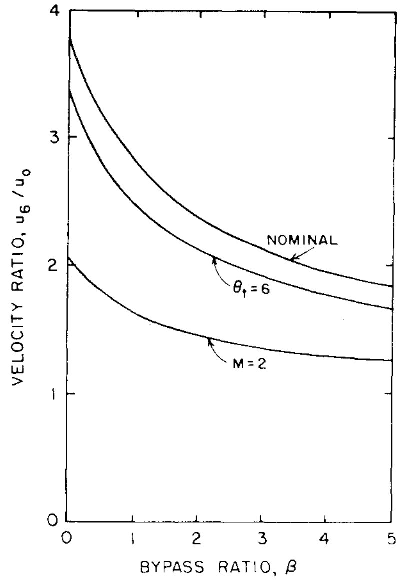

Fig. 6 Variation of engine exhaust velocity with bypass ratio.

τ_c, τ_F, τ_T are the total temperature ratios across compressor (T_{t3}/T_{t2}), fan ($T_{t3'}/T_{t2}$), and turbine (T_{t5}/T_{t4}), respectively.

Thus the temperature ratios τ are related to each other and the chosen θ's by

$$\theta_t(1-\tau_T) = \theta[(\tau_c - 1) + \beta(\tau_F - 1)] \tag{11}$$

Note that for an isentropic compressor, the overall (compressor) pressure ratio (OPR) is $\tau_c^{3.5}$, so that τ_c may be considered to be a measure of the compressor pressure ratio.

The pressure balance in the mixer can be written as

$$\tau_F = \tau_c \tau_T \tag{12}$$

Combining these allows solution for the turbine temperature ratio

$$\tau_T = [\theta_t - \theta(\tau_c - 1) + \theta\beta]/(\theta_t + \theta\tau_c\beta) \tag{13}$$

The total temperature of the mixed exhaust gas is given by an energy balance for the mixer:

$$T_{t6}/T_0 = \{\theta_t + \theta[\beta - (\tau_c - 1)]\}/(1+\beta) \tag{14}$$

The radiative characteristics of the gas are determined by the static temperature of the gas which may be determined from:

$$T_{t6}/T_6 = 1 + \tfrac{1}{2}(\gamma - 1)M_6^2 = \theta\tau_c\tau_T \tag{15}$$

or

$$T_6/T_0 = (\theta_t/\theta\tau_c + \beta)/(1+\beta) \tag{16}$$

The variation of T_6 with bypass ratio is shown in Fig. 5 for a set of values of the parameters θ_t, θ, and τ_c. A nominal case is noted, with variations of the individual parameters shown to display the sensitivity to the magnitude of the chosen values. Note the rapid decrease in T_6 when bypass ratio β is increased from zero to near unity whereafter the decrease in T_6 is much smaller for increasing β.

The jet exhaust velocity determines the thrust per unit air flow rate and the length of the plume core. For proper expansion conditions of the gas, the thrust is given by

$$F = \dot{m}_{tot}(u_6 - u_0)$$

where $\dot{m}_{tot}$ is the combined primary and fan air mass flow rate. Thus

$$F/\gamma p_0 A_6 M_0^2 = (M_6^2/M_0^2)(1-u_0/u_6) \tag{17}$$

Here M_6 is given by Eq. (15) and the ratio of u_6/u_0 may be shown to be

$$\frac{u_6}{u_0} = \frac{M_6}{M_0}\sqrt{\frac{T_6}{T_0}} = \sqrt{1+\frac{(\theta\tau_c-1)(\theta_t/\theta\tau_c-1)}{(\theta-1)(1+\beta)}} \tag{18}$$

with the aid of Eqs. (15) and (16). Figure 6 shows the resulting variation of this velocity ratio for the same conditions noted on Fig. 5. The ratio is seen to decrease with increasing bypass ratio, most rapidly near $\beta = 0$.

The variation with the overall pressure ratio, or equivalently $\tau_c = (\text{pressure ratio})^{\gamma-1/\gamma}$, is not shown in Fig. 6 because the curves for $\tau_c = 2$ and 2.5 are nearly coincident, which results from the fact that they lie on either side of the τ_c required for maximum u_6, i.e., $\tau_{cm} = \sqrt{\theta_t/\theta}$ (=2.24 in this case).

The thrust parameter given by Eq. (17) is written so that only flight conditions and A_6 are used to nondimensionalize the thrust. All cycle parameters are involved in the nondimensional right-hand side of Eq. (17), which are easily determined using Eqs. (15) and (18). Thus one obtains

$$\frac{F}{\alpha p_0 M_0^2 A_6} = \frac{(\theta\tau_c-1)(\theta\tau/\theta\tau_c-1)+(\theta-1)(1+\beta)}{(\theta-1)(\theta\tau/\theta\tau_c+\beta)} \times\left[1-\left(1+\frac{(\theta\tau_c-1)(\theta\tau/\theta\tau_c-1)}{(\theta-1)(1+\beta)}\right)^{-1/2}\right] \tag{19}$$

This expression will be useful for relating the engine thrust to its emitted radiation.

IV. Radiation Emission

The emitted infrared radiation is related to the number of CO_2 molecules in the exhaust gas which depends on the fuel flow rate. This flow rate is also important from the design point of view in that it is crucial for determining the ability of the airplane to achieve its mission. In the following, we will develop an expression for the infrared emissions which will be seen to be intimately related to the fuel consumption.

The fuel-to-air flow rate ratio is given by an energy balance across the combustor. This yields

$$\frac{\dot{m}_{fuel}}{\dot{m}_{tot}} = \frac{C_p T_0}{h}\left(\frac{\theta_t-\theta\tau_c}{1+\beta}\right) \tag{20}$$

where h is the heating value of the fuel. For varying engine cycles, this mass flow rate ratio may be used to calculate the density of the CO_2 molecules in the exhaust stream. This density is proportional to

$$\rho_{CO_2} = C\frac{\dot{m}_{fuel}}{u_6 A_6} = C\frac{\dot{m}_{tot} C_p T_0}{u_6 A_6 h}\left(\frac{\theta_t-\theta\tau_c}{1+\beta}\right) \tag{21}$$

where C is a proportionality constant related to the carbon content of the fuel. From Eq. (21) it is evident that large u_6, A_6, and small $\dot{m}_{fuel}$ are required to minimize the CO_2 density. Since $\dot{m}_{tot}$ is related to thrust we may write

$$\rho_{CO_2} = C\frac{C_p T_0}{h}\frac{F}{(u_6-u_0)}\frac{1}{u_6 A_6}\left(\frac{\theta_t-\theta\tau_c}{1+\beta}\right) \tag{22}$$

When the optical thickness of the gas is small, the radiative emission power (J, W/sr-μ) is proportional to the product of 1) emitted power per CO_2 molecule in the wavelength band of interest, 2) number of CO_2 molecules per unit volume, and 3) core volume. The CO_2 emissive power may be taken to be proportional to $(T_6/T_0)^n$ where n is treated parametrically. The CO_2 density is given by Eq. (22) and the volume of the hot core is

$$V = \frac{(u_6/u_0+1)A_6^{3/2}}{(u_6/u_0-1)Æ^{3/2}}\left[\frac{1}{3}+\frac{Æ-1}{2}\right] \tag{23}$$

using Eq. (4) applied to the geometry depicted in Fig. 1. Omitting the proportionality constants, we obtain

$$\frac{Ju_0^2}{F} \sim \left(\frac{T_6}{T_0}\right)^n \frac{u_6/u_0+1}{u_6/u_0-1}\frac{u_0}{u_6}A_6^{1/2}f(Æ)\frac{\theta_t-\theta\tau_c}{1+\beta} \quad \text{(THIN)} \tag{24}$$

where $f(Æ) = Æ^{-3/2}[1/3 + 1/2(Æ-1)]$ which is plotted in Fig. 7 and is seen to decrease from $Æ = 1$.

The emitted radiation is seen from Eq. (24) to be proportional to thrust and the proportionality depends on the ratio u_6/u_0 (a measure of specific thrust), T_6/T_0 and engine design variables. These may be readily substituted into Eq. (24). The result is plotted in Fig. 8 with $f(Æ) = 1$ and $A_6 = 1$. From this variation it is evident that the optically thin radiation decreases rapidly from $\beta = 0$ to a minimum which is

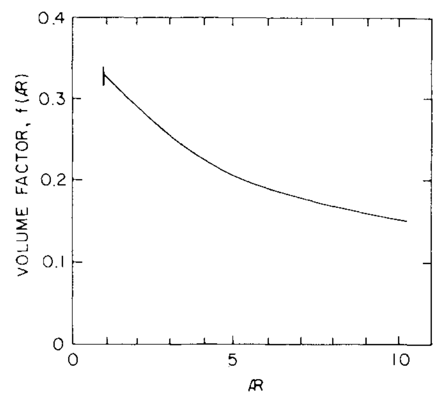

Fig. 7 Core volume variation with nozzle aspect ratio.

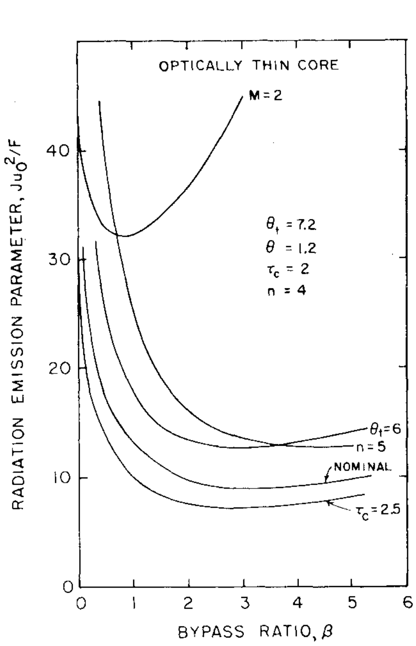

Fig. 8 Radiation emission from an optically thin exhaust core of an engine with bypass ratio β.

in the range $\beta = 1\text{-}3$, depending primarily on the flight Mach number M. For a given thrust level, the bypass ratio for minimum J decreases with increasing β. This is consistent with other requirements for low β at high speed.[11] With u_0^2 in the radiation emission parameter, it is worthy of note that J/F at the minimum is roughly independent of u_0 (or M_0).

The increase at high β is due to the small difference $u_6\text{-}u_0$ which stretches the core and increases its volume. The variations for the thermodynamic cycle parameters θ_t, τ_c show simply that the better the cycle efficiency, the lower the heat rejected and therefore the i.r. signature.

The sensitivity to the temperature exponent, n, is shown by comparing the nominal case ($n = 4$) to a case with $n = 5$, which is closer to realistic when one considers the thermal emission in the 4μ spectral range at the temperatures of interest.

The optically thick portions of the emission spectrum must be handled differently from the thin portion because the core surface area, rather than the volume, determines the source radiant intensity. It is evident that the areas seen by observers of the s and v views are, respectively, $\frac{1}{2}H_0L$, $(W_0 - \frac{1}{2}H_0)L$ where L is the core length. As shown in Eq. (10) the radiation observed by the v-view observer is larger than that seen in the s-view by a factor $(2Æ - 1)$. Restricting attention to the s-view, one can show that the source intensity is proportional to T_6^n and the area H_0L. Thus with Eq. (4)

$$J \sim \left(\frac{T_6}{T_0}\right)^n H_0^2 \frac{u_6/u_0 + 1}{u_6/u_0 - 1} \tag{25}$$

where $H_0^2 = A_6/Æ$. From this it is evident that the s-view benefits from a high aspect ratio nozzle design $[f_s(Æ) = 1/Æ]$, while the v-view suffers a penalty, $[f_v(Æ) = 2 - 1/Æ]$. This result was noted earlier in connection with the development of Eq. (10).

Omitting further discussion of Æ, we obtain an expression for Ju_0^2/F by dividing Eq. (25) by Eq. (17):

$$\frac{Ju_0^2}{F} \sim \left(\frac{T_6}{T_0}\right)^{n+1} \frac{(1 + u_0/u_6)}{(u_6/u_0 - 1)^2} \qquad \text{(THICK)} \tag{26}$$

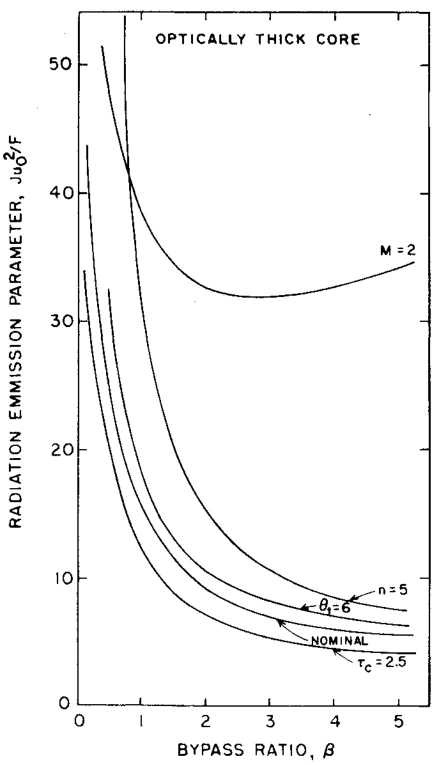

Fig. 9 Radiation emission from an optically thick exhaust core of an engine with bypass ratio β and aspect ratio unity.

This result is independent of A_6 [unlike the "thin" result, Eq. (24)] and involves only the cycle parameters through T_6 and u_6. The result is plotted in Fig. 9. Here, the parameter variation is similar to that noted in the discussion relating to Fig. 8. By way of differences, we see that the minima are seen to occur at higher values of bypass ratio.

Figures 8 and 9 are not directly comparable because of the scaling constants that appear in both ordinates. Rather, they indicate, that for a given optical situation, how variations in engine design variables influence the emitted radiation. Note also that Æ scaling and A_6 scaling differ in the two cases.

It is a relatively straightforward matter to determine the specific fuel consumption, S, of this ideal engine using Eqs. (17) and (20). This is

$$S \sim 1/u_0 (u_6/u_0 - 1)^{-1} (\theta_t - \theta\tau_c)/(1 + \beta) \tag{27}$$

It is instructive to note the variation of the radiation from thick and thin cores with S, since both decrease with increasing bypass ratio. This variation is shown in Fig. 10 for the nominal case and $n = 4$. Both variations and S were normalized to unity for the turbojet ($\beta = 0$) case. In the range $\beta = 0$ to nearly 1, the reduction in i.r. emission is nearly linear, falling to 30% of the pure turbojet case. Further reductions may be made by means of a high (10) Æ nozzle amounting to 50% (Fig. 7) for the optically thin portions of the core spectrum or 90% for the thick portions (s-view, $f_s = 1/Æ$). For cases falling between thick and thin, i.e., optically grey, results falling between these limits should be expected.

V. Afterburning

To compensate for the specific thrust loss associated with increasing bypass ratio, it may be appropriate to carry out afterburning in the mixed exhaust flow. In the following, we will develop the change in radiative power for the optically thin and thick cases, assuming a reference engine of given design is operated with a small amount of afterburning to increase thrust.

Afterburning involves the addition of heat (fuel) between state 6 and a new state 7 not shown in Fig. 4. Denoting as τ_{ab}, the nondimensional heat addition in the afterburner, we may write

$$T_{t7}/T_0 - T_{t6}/T_0 = \tau_{ab}/(1 + \beta) \tag{28}$$

If we describe by subscript 7, the static state in the jet with afterburning and by subscript 6 the static state without, we

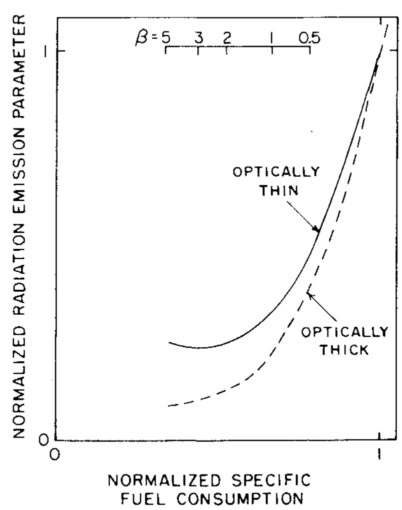

Fig. 10 Radiation emission and specific fuel consumption normalized to $\beta = 0$, for limiting thin and thick cases.

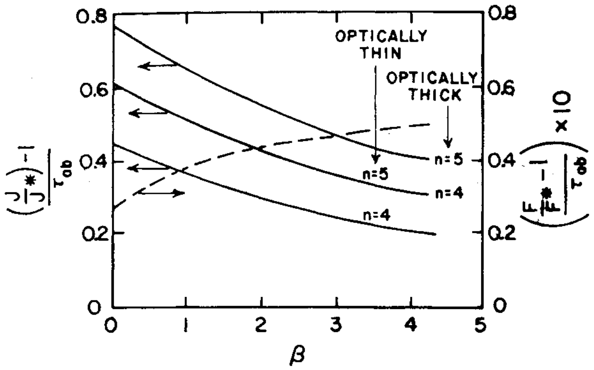

Fig. 11 Radiation and thrust augmentation by afterburning in a mixed flow turbofan of bypass ratio β.

may write

$$\frac{T_7}{T_6}=1+\frac{\tau_{ab}}{\theta_t+\theta[\beta-(\tau_c-1)]}\equiv 1+\frac{\tau_{ab}}{\Delta} \tag{29a}$$

where $\Delta=(1+\beta)\,T_{t6}/T_0$.

Since $M_7=M_6$ in the ideal engine, it follows that

$$u_7/u_6=(1+\tau_{ab}/\Delta)^{1/2} \tag{29b}$$

If we assume light afterburning, i.e., $\tau_{ab}/\Delta \ll 1$, the resulting thrust is given by

$$F/F^* \cong 1+(\tau_{ab}/2\Delta)\,(u_6/u_0-1)^{-1} \tag{30}$$

where F^* is the thrust with no afterburning. For the optically thin and thick cores we obtain with Eqs. (24) and (26), expressions for J/J^* for the two cases

THIN:

$$\frac{J}{J^*}\cong 1+\frac{\tau_{ab}}{2\Delta}\left(2n-\frac{1}{u_6/u_0+1}-\frac{2}{1-u_0/u_6}+\frac{1}{u_6/u_0-1}\right) \tag{31}$$

where the terms in the brackets give the sensitivity of radiation to afterburning and arise from the $(T_7/T_0)^n$, $(1+u_0/u_7)$, $(u_7/u_0-1)^{-2}$, and F_7/F_6 terms in Eq. (24), respectively. For the optically thick case, the result is identical except for the $2n$ term, which is found to be $2(n+1)$.

In these expressions u_6/u_0 summarizes the cycle parameter (θ_t, θ, τ_c, β) influences which could be written out by substitution of Eq. (18). Since the brackets are functions of u_6/u_0 and n only, it is possible, for the nominal case of Figs. 8-10, to plot the coefficient of τ_{ab} as a function of β. Figure 11 shows the variation of the radiation augmentation parameter $(F/F^*-1)/\tau_{ab}$ for the nominal case. From this it is evident that for a 5% thrust increase for a $\beta=1$ engine, $\tau_{ab}=1.31$ is required. $[(1.05-1)10/\tau_{ab}=0.38]$. For the central line (i.e., optically thick, $n=4$) the emitted radiation is increased by $J/J^*=1.65$. This shows the very strong influence of afterburning on i.r. signature and thus identifies the limits of its usefulness. Note that for greater thrust augmentation, i.e., $\tau_{ab}>1.3$, the simplifying linearization cannot be used and use must be made of Eqs. (24) and (26) with Eqs. (17), (28), and (29).

VI. Conclusions

The fundamental and persistent source of infrared radiation from a turbofan engine, i.e., the unmixed core of exhaust gas, is examined from the point of view of assessing the influence of nozzle aspect ratio and engine design variables. The approach is to examine the radiative emission from a perspective where only the hot core is important, and using a simple descriptive model of that core and the engine that supplies the gas. This makes the approach useful for preliminary design purposes where simple and fast tools must be used to identify configurations which are to be analyzed subsequently in greater detail.

By using a simple qualitative model to describe the radiative characteristics of the core, and fixing the cycle parameters, the effect of aspect ratio is examined. One may conclude that increasing the nozzle aspect ratio decreases the emitted radiation for those parts of the spectrum where the optical thickness is small because the core volume is reduced. For the portions of the spectrum where the optical thickness is large, a reduction is experienced only where the core is viewed along its larger dimension. The radiation in the other view is increased modestly. Thus one might expect a reduction in i.r. plume signature when designing with increasing aspect ratio nozzles, particularly when the plume is viewed along the larger dimension.

The influence of cycle variations is examined by assuming a T^n dependence of emitted radiation on temperature and a simple mixing flow model to determine core length. The exhaust temperature and velocity are calculated for an ideal cycle for which the calculation is sufficiently simple to enable the drawing of the following conclusions: Cycle variations which improve the thermal efficiency of the engine reduce i.r. emissions. Bypass ratios near unity are very effective in reducing i.r. emissions for both optically thin and thick radiation models and presumably for intermediate thicknesses. Afterburning increases the i.r. emissions ten times faster than the thrust and is thus unattractive as a cycle variation.

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